SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) UNCLASSIFIED

AD A089841

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. AD-A089841	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) EFFICIENT ACCESSING OF A MULTIACCESS CHANNEL		5. TYPE OF REPORT & PERIOD COVERED Paper-Technical
		6. PERFORMING ORG. REPORT NUMBER LIDS-P-1040
7. AUTHOR(s) Pierre A. Humblet and Jeannine Mosely		8. CONTRACT OR GRANT NUMBER(s) ARPA Order No. 3045/5-7-75 ONR/N00014-75-C-1183
9. PERFORMING ORGANIZATION NAME AND ADDRESS Massachusetts Institute of Technology Laboratory for Information and Decision Systems Cambridge, Massachusetts 02139		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Code No. 5T10 ONR Identifying No. 049-383
11. CONTROLLING OFFICE NAME AND ADDRESS Defense Advanced Research Projects Agency 1400 Wilson Boulevard Arlington, Virginia 22209		12. REPORT DATE September 1980
		13. NUMBER OF PAGES 4
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Office of Naval Research Information Systems Program Code 437 Arlington, Virginia 22217	LEVEL II	15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

DTIC SELECTE OCT 2 1980 S D C

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Multiaccess Channel Distributed Control

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A discrete time multiaccess channel is considered where the outcome of a transmission is either "idle", "success", or "collision", depending on the number of users transmitting simultaneously. Messages involved in a "collision" must be retransmitted. An efficient access allocation policy is developed for the case where infinitely many sources generate traffic in a Poisson manner and can all observe the outcomes of the previous transmissions. Its rate of success is .488. Modifications are presented for the cases where the transmission times depend on the transmission outcomes and where observations are noisy.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

80 10 1 023

To be presented at the 19th IEEE Conference on Decision and Control, Dec. 10-12, 1980, in Albuquerque, New Mexico

(11) Sept[redacted]80 (9) Technical paper, (14) LIDS-P-1040

Dec. and Control.

(12) 7

(6) EFFICIENT ACCESSING OF A MULTIACCESS CHANNEL*

(10) Pierre A./Humblet [redacted] Jeannine/Mosely
Laboratory for Information and Decision Systems
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Abstract

A discrete time multiaccess channel is considered where the outcome of a transmission is either "idle", "success", or "collision", depending on the number of users transmitting simultaneously. Messages involved in a "collision" must be retransmitted. An efficient access allocation policy is developed for the case where infinitely many sources generate traffic in a Poisson manner and can all observe the outcomes of the previous transmissions. Its rate of success is .488. Modifications are presented for the cases where the transmission times depend on the transmission outcomes and where observations are noisy.

I. Introduction

We consider the following model of a multiple access channel. A large number of sources generate messages in a Poisson manner, at a total rate of λ messages per unit of time, starting at time 0. Once a message has been generated, its source can transmit it on a common channel. Transmissions can only start at integer multiples of the unit of time and last one unit of time. If the transmissions from two or more sources overlap, a collision is said to occur, all messages are lost and must be retransmitted at a later time. If only one source transmits, the transmission is successful.

All sources can observe the channel and learn instantaneously whether it is idle, or if a success or a collision has occured. This common feedback is the only information the sources share. The problem is to find an effective way of using the feedback to schedule the transmission of the messages.

The previous model is an idealization of practical communication systems [1],[2],[3] that have been the object of numerous papers in the communication theory literature [4],[5]. Similar problems have also been treated in control theory journals [6],[7],[8], indeed they are nice examples of distributed control.

The next section will present the basic algorithm and some of its properties. This will be followed by an analysis and optimization. We then show how to modify and analyze the algorithm if the transmission times depend on the transmission outcomes or if the feedback is noisy.

II. The Basic Algorithm

The algorithm defined below allows the transmissions of the messages on the basis of their generation times, rather than on the basis of the identities of their sources. It has the advantage of being effective no matter the number of sources, even infinite, and is a generalization of the procedure presented in [9].

*This research was conducted at the M.I.T. Laboratory for Information and Decision Systems with partial support provided by grant DARPA/ONR-N00014-065-C-1183 and grant NSF/ECS-79-19880.

(15) N00014-75-C-1183, ARPA Order-3645

The set of messages that are transmitted during the nth time unit interval are those generated in the time interal $[y_n, y_n+F(s_n,t_n))$. F is a given function (to be optimized below) mapping $\mathbb{R}^+\cup\{\infty\} \times \mathbb{R}^+\cup\{\infty\}$ into $\mathbb{R}^+$, with the property that $F(s,t) < t$. y_o, s_o and t_o are initially equal respectively to o, ∞ and ∞, and the y_n's, s_n's and t_n's ($s_n \le t_n$) are updated by the following rule, where $F_n = F(s_n,t_n)$. If a transmission results in

$$\text{idle}: \begin{cases} y_{n+1} \leftarrow y_n + F_n \\ s_{n+1} \leftarrow s_n - F_n \\ t_{n+1} \leftarrow t_n - F_n \end{cases}$$

$$\text{success}: \begin{cases} y_{n+1} \leftarrow y_n + F_n \\ s_{n+1} \leftarrow t_n - F_n \\ t_{n+1} \leftarrow \infty \end{cases}$$

$$\text{collision}: \begin{cases} y_{n+1} \leftarrow y_n \\ s_{n+1} \leftarrow \min[s_n, F_n] \\ t_{n+1} \leftarrow F_n \end{cases}$$

Given the outcomes of the n past transmissions the following can be said about the message generation times:

1. all messages generated in $[0,y_n)$ have been successfully transmitted

2. the message generation times in $[y_n+t_n,\infty]$ are distributed according to a Poisson process with rate λ, and are independent of the generation times in $[0,y_n+t_n)$

3. the message generation times in $[y_n, y_n+t_n)$ are independent of the message generation times in $[0,y_n)$ and $[y_n+t_n,\infty]$, and are distributed according to a Poisson process with rate λ, conditioned on the facts that there are at least one generation time in $[y_n,y_n+s_n)$ and two generation times in $[y_n,y_n+t_n)$.

To prove those assertions, note that they are true initially (n=0) and remain true as n increases. Checking them all is simple, but a rigorous proof would take much space; we will only sketch the method by treating one case. Details can be found in [10]. Examine Figure 1 below.

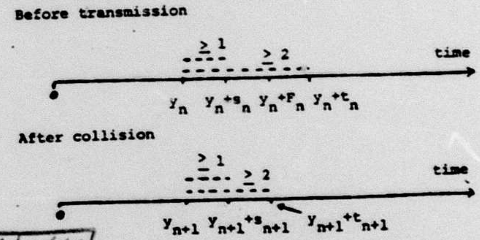

Figure 1: example

Accession For	
NTIS GRA&I	✓
DTIC TAB	☐
Unannounced	☐
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A	

410950

Assume $s_n \leq F_n < t_n$, and the transmission results in a collision. Assertion 1 remains true as $y_{n+1} = y_n$. To verify assertions 2 and 3, note that due to the Poisson nature of the generation process the numbers of message generations in $[y_n, y_n+F_n)$ and $[y_n+F_n, y_n+t_n)$ are independent , and that there are at least two in the first interval implies that there are at least two in the union of the two intervals. Thus the new feedback information that there are at least two generation times in $[y_n, y_n+F_n)$ makes obsolete the old information that there are at least two in $[y_n, y_n+t_n)$, and the only information we have about $[y_n+F_n, y_n+t_n)$ is the knowledge of its a priori statistics.

Before proceeding with the next section which will show how to define $F(\cdot,\cdot)$ so as to maximize the rate at which messages are successfully transmitted, we will make two remarks.

First, in its form just given, the algorithm is not causal, in the *sense* that it sometimes specifies that messages should be transmitted before having been generated. This can be remedied to by defining $F_n = \min\,[F(s_n,t_n),\ n-y_n]$. However, for the purpose of this paper, we will keep the original form, as we are only interested in the maximum rate at which messages can be successfully transmitted, and not in real time properties, like message waiting times until successful transmission.

Second, this algorithm is the most general algorithm that guarantees that messages are transmitted in *the order they were generated* (a desirable fairness property), although it is far from being the most general access algorithm.

III. Analysis and Optimization

The key to the analysis of the algorithm is to realize that the process (s_n, t_n) is Markovian, as the probabilities of the different outcomes of the (n+1)th transmission and of the values of (s_{n+1}, t_{n+1}) depend only on s_n and t_n. Thus in the case illustrated in the top part of Figure 1, the value of (s_{n+1}, t_{n+1}) will be (t_n-F_n, ∞) in case of success (i.e., with probability Pr[1 Poisson arrival in $[y_n, y_n+F_n)$ | at least 1 Poisson arrival in $[y_n, y_n+s_n)$ and at least 2 in $[y_n, y_n+t_n)$]). In case of collision the value of (s_{n+1}, t_{n+1}) would be (s_n, F_n). The channel cannot remain idle in this case as $s_n \leq F_n$.

It is straightforward but tedious to write down the transition probabilities for all cases. We should notice the peculiar role of the (∞,∞) state. Physically it corresponds to all messages generated before y_n having been successfully transmitted and no information except the a priori statistics being available about generation times greater than y_n. That state is entered at least every time two consecutive transmissions result in a success, thus it is reachable from all other states.

Moreover if $F(\cdot,\cdot)$ is such that there is a positive lowerbound on the probability of successful transmission in any state (s,t) (this is always the case for the $F(\cdot,\cdot)$'s considered below), then state (∞,∞) is positive recurrent along with only countably many other states accessible from it. Thus the computation of state probabilities and expected values, with a given degree of precision, is a straightforward numerical matter.

We will now direct our attention to the problem of selecting $F(\cdot,\cdot)$ to maximize the long term rate of success (also called throughput), i.e.,

$\lim_{N\to\infty} \frac{1}{N} E[\sum_{i=0}^{N} r(i)]$, where r(i) is equal to one if the ith transmission is successful, and zero otherwise.

This can be done simply by discretizing the state space and using the successive approximation method [11] of solving undiscounted infinite horizon Markovian decision theory problems.

The details of the work appear in [10]. The following conclusions were reached

b) the optimal F(s,t) is never greater than s, so that all states (s,t) with $s \neq t$ or $t \neq \infty$ are transient.

c) the optimal $F(\infty,\infty)$ is $1.26/\lambda$ so that all states (s,t) with $\infty > t \geq s > 1.26/\lambda$ are transient.

d) the optimal F(s,s), $s \leq 1.26/\lambda$, is very close to s/2. In fact the throughput of the algorithm [9] using $F(\infty,\infty) = 1.26/\lambda$, $F(s,s) = \frac{s}{2}$, $(s<\infty)$ and $F(s,\infty) = s$ $(s<\infty)$ is .487. This last algorithm is itself a generalization of the tree algorithm [12] which introduced binary splitting.

Note however that remark b) above does not hold for finite horizon (finite N) problems, where the optimal $F(s,\infty)$ at time N may be larger than s for $N \geq 3$ and s below some threshold, and has a large discontinuity at the threshold. The threshold decreased with N, becoming smaller than the grid size ($.01/\lambda$) for $N \geq 5$. No similar behavior was observed for F(s,t), $t<\infty$, probably because of the numerical optimization did not consider (transient) states in the region where the phenomenon would occur.

IV. Unequal Observation Times

In fact many multiaccess communication systems differ from the model introduced in section 1 in that the times necessary to learn the transmission outcomes depend on the outcomes. We denote by t_o, t_1 and t_2 respectively the times necessary to learn that the channel was idle, or that a success or a collision occured.

For example carrier sense radio systems [2] can detect idles quickly (no carrier present), while they rely on error detecting codes and the transmissions of acknowledgements to distinguish between successes and collisions, thus $t_o \ll t_1 = t_2$. In addition, some cable broadcast systems [3] have a listen-while-transmit feature that allows the quick abortion of transmissions resulting in collisions, thus $t_o = t_2 \ll t_1$.

The general algorithm outlined in section 2 and the remarks about its Markovian nature remain valid, but the reward function $r(\cdot)$ and the maximization in section III are not appropriate. A better measure of quality is to minimize the expected time to send a message, i.e.,

$$\lim_{N\to\infty} E \frac{\sum_{i=1}^{N} \sum_{j=1}^{3} t_j \, I(m(i) = j)}{\sum_{i=1}^{N} r(i)} = t_1 + t_2 \lim_{N\to\infty} E \frac{\sum_{i=1}^{N} \frac{t_o}{t_2} I(m(i) = 0) + I(m(i) = 2)}{\sum_{i=1}^{N} r(i)}$$

where $m(i)$ is 0,1,2 depending on the outcome of the ith transmission, and $I(\cdot)$ denotes the indicator function.

The limit of the expected value in the right hand side can be interpreted as the expected time overhead per message, and depends only on t_o/t_2 for a given $F(\cdot,\cdot)$. It will be denoted by c and should be minimized over $F(\cdot,\cdot)$ for a given t_o/t_2.

The optimization of the general algorithm under this formulation is time consuming. It is greatly simplified if we consider only those $F(\cdot,\cdot)$ such that $F(s,t) \leq s$. The only recurrent states are then of the form (s,s), or (s,∞), see above. Note that the optimal F found in section III belonged to the restricted class. We will now show how to proceed with the optimization.

By a renewal argument

$$c = \frac{E \sum_{i=1}^{b} \frac{t_o}{t_2} I(m(i) = 0 + I(m(i) = 2)}{E \sum_{i=1}^{b} r(i)}$$

where in the right hand side one assumes that $(s_1, t_1) = (\infty,\infty)$, and b is the time of first return at (∞,∞).

Let us now assume that we guess a value $\hat{c}$ for the minimum of c over all restricted $F(\cdot,\cdot)$, and consider the function

$$V(s,t) = E\left(\sum_{i=1}^{b} \left(\frac{t_o}{t_2} I(m(i) = 0) + I(m(i) = 2) - \hat{c}\, r(i)\right) \,\middle|\, (s_1, t_1) = (s,t)\right)$$

Because s_{n+1} is either equal to ∞ or is less than s_n, $V(s,s)$ and $V(s,\infty)$ can be written as convex combinations of $V(s',s')$ and $V(s',\infty)$, $s' < \min(s, F(\infty,\infty))$. It is straightforward [10] to minimize $V(s,s)$ and $V(s,\infty)$ recursively for increasing s, and to obtain the minimum value of $V(\infty,\infty)$.

If the minimum value is 0, $\hat{c}$ was guessed correctly and is the minimum value of c. If the minimum value of $V(\infty,\infty)$ is positive (negative), $\hat{c}$ was guessed too small (large), and the minimization of $V(\cdot)$ must be repeated with a new $\hat{c}$.

The resulting minimum value of c is shown in Figure 2, as a function of t_o/t_2. It is almost equal to the expected time overhead per message for the algorithm where the value of $F(s,s)$ is taken as $s/2$, $s < \infty$, the value of $F(s,\infty)$ as s $s < \infty$, and only $F(\infty,\infty)$ is optimized.

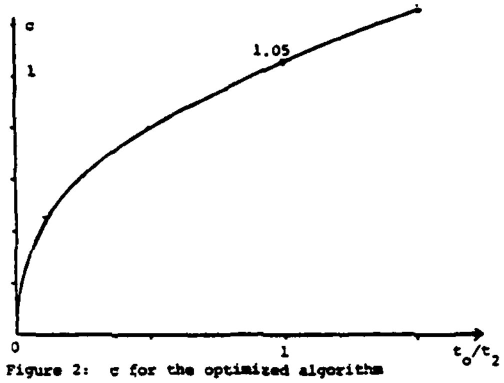

Figure 2: c for the optimized algorithm

V. Noisy Feedback

The previous algorithm assumed that the transmission outcomes were perfectly observed by all sources. This assumption is critical. One verifies easily that if an idle is falsely observed as a collision then the algorithm will deadlock, i.e., $y_n + t_n$ will remain constant, while the t_n's will decrease to zero.

D. Ryter [13] has recently examined the problem of noisy feedback, where the noise can cause idles or successes to be observed as collisions. He showed that the binary splitting algorithm [9] outlined in section III can be modified to work properly. The essential modification is the introduction of a threshold value. If t_n is smaller than the threshold, then the algorithm becomes non stationary, in the sense that if alternates between using $F(s,s) = s$ and $F(s,s) = s/2$, thus first seeking confirmation that a collision really occured, then trying to resolve it. The analysis and optimization are too long to be reported here. The main result is that with the proper choice of parameters, the throughput behaves roughly like $.487 - p$, where p is the probability of false collision indication.

VI. Final Comments

The main results of this paper are the description and analysis of an access algorithm for the channel model described in section 1, with infinitely many sources. Its throughput is .488, the largest known to this day. Molle [14] has recently shown that no algorithm can have a throughput higher than .67, and it is widely believed that the best achievable throughput is in the neighborhood of .5. However, throughputs arbitrarily close to 1 are possible, at the expense of high average message delay, when the number of sources is finite.

We have also shown how the algorithm can be modified in the cases of variable transmission times and noisy feedback.

Finally it should be pointed out that although the algorithm presented here uses the message generation times to specify when they should be transmitted, this is not necessary. Another algorithm can be described, with the same throughput and expected time overhead per message, where sources generate random numbers to determine if they should transmit. Of course real time properties, like first-generated-first-transmitted will not be conserved.

Acknowledgement: We wish to thank Mr. Joseph Hui for performing the numerical optimization that led to Figure 2.

References

[1] Roberts, L.G., "Aloha Packet System With and Without Slots and Capture", ASS, Note 8, June 26, 1972.

[2] Kleinrock, L. and Tobagi, F., "Carrier Sense Multiple Access for Packet Switched Radio Channels", Proc. Int. Conf. Communications, Minneapolis, Minn. June 1974.

[3] Metcalfe, R.M. and Boggs, D.R., "Ethernet: Distributed Packet Switching for Local Computer Networks", CACM, July 1976.

[4] Kleinrock, L. and Yemini, Y., "An Optimal Adaptive Scheme for Multiple Access Broadcast Communication", Proc. Int. Conf. Commun., pp.7.2.1-7.2.5, 1978.

[5] Kleinrock, L. and Lam, S.S., "Packet Switching in a Multiaccess Broadcast Channel: Performance Evaluation", IEEE Trans. Commun., Vol. COM-23, pp. 410-422, April 1975.

[6] Schoute, F., "Decentralized Control in Packet Switched Satellite Communication", IEEE Transactions on Automatic Control, Vol. AC-23, No. 2, April 1978.

[7] Varaiya, P. and Walrand, J., "Decentralized Control in Packet Switched Satellite Communication", IEEE Transactions on Automatic Control, Vol. AC-24, No. 5, October 1979.

[8] Varaiya, P. and Walrand, J., "A Minimum Principle for Dynamic Teams", presented at the Third Kingston Conference on Differential Games and Control Theory, Univ. of Rhode Island, June 1978.

[9] Gallager, R.G., "Conflict Resolution in Random Access Broadcast Networks", Proceedings AFOSR Workshop in Communication Theory and Applications, Sept. 17-20, 1978, Provincetown, Mass. pp. 74-76.

[10] Mosely, J., "An Efficient Contention Resolution Algorithm for Multiple Access Channels", Report LIDS-TH-918, Laboratory for Information and Decision Systems, M.I.T., May 1979.

[11] Bertsekas, D., "Dynamic Programming and Stochastic Control", Academic Press, 1976.

[12] Capetanakis, J., "The Multiple Access Broadcast Channel: Protocol and Capacity Considerations", Ph.D. Thesis, Dept. of Electrical Engineering and Computer Science, M.I.T., 1977; also IEEE Trans. I.T., Vol. IT-25, pp. 505-515, 1979.

[13] Ryter, D.M., "A Conflict Resolution Algorithm for Noisy Multiaccess Channels", Report LIDS-TH-1007, Laboratory for Information and Decision Systems, M.I.T., June 1980.

[14] Molle, M.L., "On the Capacity of Infinite Population Multiple Access Protocol", Computer Science Department, UCLA, 1980.

Distribution List

Defense Documentation Center Cameron Station Alexandria, Virginia 22314	12 Copies
Assistant Chief for Technology Office of Naval Research, Code 200 Arlington, Virginia 22217	1 Copy
Office of Naval Research Information Systems Program Code 437 Arlington, Virginia 22217	2 Copies
Office of Naval Research Branch Office, Boston 495 Summer Street Boston, Massachusetts 02210	1 Copy
Office of Naval Research Branch Office, Chicago 536 South Clark Street Chicago, Illinois 60605	1 Copy
Office of Naval Research Branch Office, Pasadena 1030 East Greet Street Pasadena, California 91106	1 Copy
Naval Research Laboratory Technical Information Division, Code 2627 Washington, D.C. 20375	6 Copies
Dr. A. L. Slafkosky Scientific Advisor Commandant of the Marine Corps (Code RD-1) Washington, D.C. 20380	1 Copy

Office of Naval Research
Code 455
Arlington, Virginia 22217 — 1 Copy

Office of Naval Research
Code 458
Arlington, Virginia 22217 — 1 Copy

Naval Electronics Laboratory Center
Advanced Software Technology Division
Code 5200
San Diego, California 92152 — 1 Copy

Mr. E. H. Gleissner
Naval Ship Research & Development Center
Computation and Mathematics Department
Bethesda, Maryland 20084 — 1 Copy

Captain Grace M. Hopper
NAICOM/MIS Planning Branch (OP-916D)
Office of Chief of Naval Operations
Washington, D.C. 20350 — 1 Copy

Advanced Research Projects Agency
Information Processing Techniques
1400 Wilson Boulevard
Arlington, Virginia 22209 — 1 Copy

Dr. Stuart L. Brodsky
Office of Naval Research
Code 432
Arlington, Virginia 22217 — 1 Copy

Captain Richard L. Martin, USN
Commanding Officer
USS Francis Marion (LPA-249)
FPO New York 09501 — 1 Copy